# Magnetohydrodynamics in Presence of Electric and Magnetic charges

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**Abstract**

Starting with the generalized electromagnetic field equations of dyons, we have discussed the theory of magnetohydrodynamics (MHD) of plasma for particles carrying simultaneously the electric and magnetic charges (namely dyons). It is shown that the resultant system supports the electromagnetic duality of dyons. Consequently the frequency of dyonic plasma has been obtained and it is emphasized that there is a different plasma frequency for each species depending on wave number $k$. For $k$ to be real, only those generalized electromagnetic waves are allowed to pass, for which the usual frequency is greater than the plasma frequency (i.e. $\omega > \omega_p$). It is shown that the plasma frequency sets the lower cuts for the frequencies of electromagnetic radiation that can pass through a plasma . Accordingly the ohm's law has been reestablished to derive the plasma oscillation equation as well as the magetohydrodynamic wave equation and the energy of dyons in unique and consistent manner.

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# 1 Introduction

Magneto hydrodynamics (MHD) is a branch of the science of the dynamics of matter moving in an electromagnetic field [1] and thus provides one of the most useful fluid models, focusing on the global properties of plasma. In a series of papers, we [2, 3, 4, 5, 6, 7] have undertaken the study of dual electrodynamics and superluminal electromagnetic fields, developed the quaternionic formulation of dyons in isotropic homogeneous, chiral and inhomogeneous media and obtained the solutions for the classical problem of moving dyon in unique and consistent way. Coceal et. al. [8] derived consistently the duality invariant magnetohydrodynamics and their dyonic solutions. Keeping in view, in this paper, we have discussed the theory of magnetohydrodynamics (MHD) of plasma for particles carrying simultaneously the electric and magnetic charges (namely dyons). It is shown that the resultant system supports the electromagnetic duality of dyons. Consequently the frequency of dyonic plasma has been obtained and it is emphasized that there is a different plasma frequency for each species depending on wave number $k$. For $k$ to be real, only those generalized electromagnetic waves are allowed to pass, for which the usual frequency is greater than the plasma frequency (i.e. $\omega > \omega_p$). It is shown that the plasma frequency sets the lower cuts for the frequencies of electromagnetic radiation that can pass through a plasma . Accordingly the ohm's law has been reestablished to derive the plasma oscillation equation as well as the magetohydrodynamic wave equation and the energy of dyons in unique and consistent manner.

# 2 Fields Associated with dyons

Postulating the existence of magnetic monopoles, the generalized Dirac Maxwell's (GDM) equations [9] are expressed in SI units ($c = \hbar = 1$) as

$$
\begin{aligned}
\overrightarrow{\nabla}. \overrightarrow{E} &= \rho_e; \\
\overrightarrow{\nabla}. \overrightarrow{H} &= \rho_m; \\
\overrightarrow{\nabla} \times \overrightarrow{E} &= -\frac{\partial \overrightarrow{H}}{\partial t} - \overrightarrow{j_m}; \\
\overrightarrow{\nabla} \times \overrightarrow{H} &= \frac{\partial \overrightarrow{E}}{\partial t} + \overrightarrow{j_e};
\end{aligned}
\tag{1}
$$

where $\rho_e$ and $\rho_m$ are respectively the electric and magnetic charge densities, $\overrightarrow{j_e}$ and $\overrightarrow{j_m}$ are the corresponding current densities , $\vec{E}$ is electric field and $\vec{H}$ is magnetic field. GDM equations (1) are invariant not only under Lorentz and conformal transformations but also invariant under the following duality transformations [2, 10, 11],

$$
\begin{aligned}
\overrightarrow{E} &= \overrightarrow{E} \cos\theta + \overrightarrow{H} \sin\theta; \\
\overrightarrow{H} &= -\overrightarrow{E} \sin\theta + \overrightarrow{H} \cos\theta.
\end{aligned}
\tag{2}
$$

For a particular value of $\theta{=}\frac{\Pi}{2}$, equation (2) reduces to

$$\overrightarrow{E}\rightarrow\overrightarrow{H},\;\;\overrightarrow{H}\rightarrow-\;\;\overrightarrow{E},\tag{3}$$

which can be written as

$$\left(\begin{array}{c}\overrightarrow{E}\\ \overrightarrow{H}\end{array}\right)\Rightarrow\left(\begin{array}{cc}0 & 1\\ -1 & 0\end{array}\right)\left(\begin{array}{c}\overrightarrow{E}\\ \overrightarrow{H}\end{array}\right).\tag{4}$$

If we apply the transformation (3) and (4) along with the following duality transformations for current i.e;

$$\rho_{e}\rightarrow\rho_{m},\rho_{m}\rightarrow-\rho_{e}\Longleftrightarrow\left(\begin{array}{c}\rho_{e}\\ \rho_{m}\end{array}\right)\Rightarrow\left(\begin{array}{cc}0 & 1\\ -1 & 0\end{array}\right)\left(\begin{array}{c}\rho_{e}\\ \rho_{m}\end{array}\right).\tag{5}$$

Differential equation (1) are the generalized field equations of dyons and the corresponding electric and magnetic fields are then called generalized electromagnetic field of dyons are expressed in the following differential form in terms of two potentials [2]:

$$\overrightarrow{E}=-\overrightarrow{\nabla}\;\;\phi_{e}-\frac{\partial\overrightarrow{A}}{\partial t}-\;\;\overrightarrow{\nabla}\times\overrightarrow{B};\tag{6}$$

$$\overrightarrow{H}=-\overrightarrow{\nabla}\phi_{g}-\frac{\partial\overrightarrow{B}}{\partial t}\;\;+\;\;\overrightarrow{\nabla}\times\overrightarrow{A};\tag{7}$$

where $\{A^{\mu}\}{=}\left\{\phi_{e},\,\overrightarrow{A}\right\}$ and $\{B^{\mu}\}{=}\left\{\phi_{g},\,\overrightarrow{B}\right\}$are the two four - potentials associated with electric and magnetic charges. Let us define the complex vector field $\overrightarrow{\psi}$ in the following form,

$$\overrightarrow{\psi}=\;\;\overrightarrow{E}-i\;\;\overrightarrow{H},\tag{8}$$

equations (6,7) and (8) , thus give rise to the following relation between generalized field and the components of the generalized four - potential as;

$$\overrightarrow{\psi}=-\frac{\partial\overrightarrow{V}}{\partial t}-\overrightarrow{\nabla}\phi-i\overrightarrow{\nabla}\;\;\times\;\;\overrightarrow{V}.\tag{9}$$

Here $\{V_{\mu}\}$ is the generalized four - potential of dyons and defined as ;

$$\{V_{\mu}\}\;\;=\;\;\left\{\phi,-\overrightarrow{V}\right\};\tag{10}$$

where

$$\phi = \phi_e - \quad i \quad \phi_m; \tag{11}$$

and

$$\overrightarrow{V} = \quad \overrightarrow{A} - i \quad \overrightarrow{B}. \tag{12}$$

If we apply the transformation (4) and (5) the following duality transformation for potential is obtained i.e.

$$\overrightarrow{A} \rightarrow \overrightarrow{B}, \overrightarrow{B} \rightarrow -\overrightarrow{A} \Rightarrow \begin{pmatrix} \overrightarrow{A} \\ \overrightarrow{B} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \overrightarrow{A} \\ \overrightarrow{B} \end{pmatrix}; \tag{13}$$

$$\phi_e \rightarrow \phi_m, \phi_e \rightarrow -\phi_m \Rightarrow \begin{pmatrix} \phi_e \\ \phi_m \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi_e \\ \phi_m \end{pmatrix}. \tag{14}$$

Maxwell field equation (1) may then be written in terms of generalized field $\overrightarrow{\psi}$ as;

$$\overrightarrow{\nabla}. \quad \overrightarrow{\psi} \quad = \rho;$$

$$\overrightarrow{\nabla} \quad \times \overrightarrow{\psi} = -i\overrightarrow{J} - i \quad \frac{\partial \overrightarrow{\psi}}{\partial t}; \tag{15}$$

where $\rho$ and $\vec{J}$ are the generalized charge and current source densities of dyons and given by;

$$\rho = \rho_e \quad -i \quad \rho_g; \tag{16}$$

$$\overrightarrow{J} = \overrightarrow{j_e} \quad -i \quad \overrightarrow{j_m}. \tag{17}$$

Here, we may write the tensorial form of generalized Maxwell Dirac equation of dyons as,

$$\begin{aligned} F_{\mu\nu,\nu} &= j_\mu^e; \\ F_{\mu\nu,\nu}^d &= j_\mu^m; \end{aligned} \tag{18}$$

where

$$\{j_\mu^e\} = \left\{\rho_e, \overrightarrow{j_e}\right\} \quad and \quad \{j_\mu^m\} = \left\{\rho_m, \overrightarrow{j_m}\right\}.$$

Defining the generalized field tensor of dyons as;

$$G_{\mu\nu,\nu} = F_{\mu\nu} - i \quad F_{\mu\nu}^d \tag{19}$$

one can directly obtained the following generalized field equation of dyon i.e;

$$\begin{aligned} G_{\mu\nu,\nu} &= J_\mu; \\ G_{\mu\nu,\nu} &= 0; \end{aligned} \tag{20}$$

where

$$\{J_\mu\} = \left\{\rho, -\overrightarrow{J}\right\}.$$

The Lorentz four - force equation of motion for dyon is written as;

$$f_\mu = m_0 \ddot{x}_\mu = \quad Re \quad Q^*(G_{\mu\nu} u^\nu) \tag{21}$$

where $'Re'$ denotes the real part, $\{\ddot{x}_\mu\}$ is the four - acceleration and $\{u^\nu\}$ is the four - velocity of the particle and $Q$ is the generalized charge of dyon.

## 3 Magnetohydrodynamic (MHD) equations for dyons

Three well known equations for magnetohydrodynamics (MHD) [1] are

$$\frac{\partial n}{\partial t} + \overrightarrow{\nabla} \quad .(n\,\overrightarrow{u}) = \quad 0 \tag{22}$$

$$\frac{\partial f}{\partial t} \quad +(\overrightarrow{v}.\overrightarrow{\nabla} f) + \frac{\vec{F}}{m}\frac{\partial f}{\partial v} = \left(\frac{\partial f}{\partial t}\right)_c \tag{23}$$

$$mn\left[\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}.\overrightarrow{\nabla})\,\overrightarrow{u}\right] = en\overrightarrow{F} + \triangle\overrightarrow{P}. \tag{24}$$

Equation (22) is known as continuity equation, (23) is recalled as the Boltzmann equation while the equation (24) has been used for famous Ohm's law. In equations (22 - 24), $n$ is

the number density , $\overrightarrow{u}$ is the drift velocity (fluid velocity), $\overrightarrow{v}$ is the particle velocity, $f$ is the partition (distribution) function, $\overrightarrow{F}$ is the force acting on the particle, $m$ is the mass of the particle, $c$ is used for collision, $e$ is the electric charge and $\triangle\overrightarrow{P}$ denotes the change in momentum due to collision. Let us discuss these equations for the particles associated with the simultaneous existence of electric and magnetic charges (dyons). To start with the familiar continuity equation of hydrodynamics (22) with respect to the conservation of matter (charge) we express

$$\frac{\partial n}{\partial t}+\overrightarrow{\nabla}.(n\overrightarrow{u})=0 \tag{25}$$

For the case of electric charge, $n$ is replace by electric charge density $\rho_e$. So the equation (25) takes the following form

$$\frac{\partial\rho_e}{\partial t}+\overrightarrow{\nabla}.(\rho_e\overrightarrow{u})=0. \tag{26}$$

Similarly on postulating the existence of magnetic charge (i.e. monopole), we may replace $n$ by magnetic charge density $\rho_m$. Hence the equation (25) reduces to the continuity equation in presence of pure magnetic monopole i.e.

$$\frac{\partial\rho_m}{\partial t}+\overrightarrow{\nabla}.(\rho_m\overrightarrow{u})=0. \tag{27}$$

As such, for the case of plasma for dyon (i.e; particle carrying both electric and magnetic charge simultaneously), we may replace the number density $n$ by generalized source density $\rho=\rho_e-i\rho_g$ of dyons. So using equations (25 - 27), we get

$$\frac{\partial\rho}{\partial t}+\overrightarrow{\nabla}.(\rho\overrightarrow{u})=0 \tag{28}$$

which represents the generalized form of continuity equation (first magnetohydrodynamics (MHD) equation) for dyonic plasma.

Similarly, we may express the second equation of magnetohydrodynamics (MHD) for the plasma of dyons. For this we replace the force $\overrightarrow{F}$ of equation (23) as the force exerting on the particle simultaneously carrying electric and magnetic charges (dyons). The force of dyons given by equation (21) reduces to

$$\overrightarrow{F}=e(\overrightarrow{E}+\overrightarrow{v}\times\overrightarrow{H})+g(\overrightarrow{H}-\overrightarrow{v}\times\overrightarrow{E}). \tag{29}$$

Substituting the equation (29) for force $\overrightarrow{F}$ of dyons into equation (23) and multiplying by $m\overrightarrow{v}$ and then integrating it over the velocity $d\overrightarrow{v}$, we get,

$$m\int \vec{v}\frac{\partial f}{\partial t}d\vec{v}+m\int \vec{v}(\vec{v}.\vec{\nabla}f)d\vec{v}$$
$$+\int \vec{v}[e(\vec{E}+\vec{v}\times\vec{H})+g(\vec{H}-\vec{v}\times\vec{E})]\frac{\partial f}{\partial v}d\vec{v}=\int m\vec{v}\left(\frac{\partial f}{\partial t}\right)_c d\vec{v}. \tag{30}$$

The first term of equation (30) reduces to

$$m\int \vec{v}\frac{\partial f}{\partial t}d\vec{v}=m\frac{\partial}{\partial t}\int \vec{v}fd\vec{v}=m\frac{\partial}{\partial t}\quad(n\vec{u})\quad . \tag{31}$$

Similarly, the second term of equation (30) is expressed as,

$$m\int \vec{v}(\vec{v}.\vec{\nabla}f)d\vec{v}=m\int(\vec{\nabla}.f\vec{v})\,\vec{v}d\vec{v}=m\int(\vec{\nabla}.f\vec{v})\vec{v}d\vec{v} \tag{32}$$

where

$$\int(\vec{\nabla}.f\vec{v})\quad \vec{v}d\vec{v}=(\vec{\nabla}.\overline{n\vec{v}})\vec{v}\quad . \tag{33}$$

Expressing the particle velocity $\vec{v}$ in terms of the average velocity (fluid velocity) $\vec{u}$ and thermal velocity $\vec{w}$, as

$$\vec{v}\quad=\vec{w}+\quad\vec{u} \tag{34}$$

we get

$$(\vec{\nabla}.\overline{n\vec{v}})\vec{v}=(\vec{\nabla}.\overline{n\vec{u}})\vec{u}+(\vec{\nabla}.\overline{n\vec{w}})\vec{w}+\quad 2\quad(\vec{\nabla}.\overline{n\vec{u}})\vec{w} \tag{35}$$

where $\overline{w}$ the average thermal velocity vanishes. The quantity $m(\overline{n\vec{w}})\vec{w}$ is expressed as the stress tensor $I$. Then the first term of equation (35) reduces to

$$(\vec{\nabla}.\overline{n\vec{u}})\vec{u}\quad=\vec{u}(\vec{\nabla}.n\vec{u})+\quad n(\vec{u}.\vec{\nabla})\vec{u}. \tag{36}$$

As such, equation (32) reduces to

$$m\int \vec{v}(\vec{v}.\vec{\nabla}f)d\vec{v}=m\vec{u}\vec{\nabla}.(nu)+\quad mn(\vec{u}.\vec{\nabla})\vec{u}+\quad\vec{\nabla}.I. \tag{37}$$

So, the third term of equation (30) is expressed as

$$e\int \vec{v}(\vec{E}+\vec{v}\times\vec{H})\frac{\partial f}{\partial v}d\vec{v}=-en_e(\vec{E}+\vec{v}\times\vec{H}). \tag{38}$$

Similarly the fourth term of equation (30) reduces to

$$g \int \overrightarrow{v}(\overrightarrow{H} - \overrightarrow{v} \times \overrightarrow{E}) \frac{\partial f}{\partial t} d\overrightarrow{v} = -g n_m (\overrightarrow{H} - \overrightarrow{v} \times \overrightarrow{E}). \tag{39}$$

On the other hand the right hand side term of equation (30) is analogous to the change in momentum $\triangle \overrightarrow{P}$ due to collision i.e;

$$\triangle \overrightarrow{P} = \int mv \left( \frac{\partial f}{\partial t} \right)_c d\overrightarrow{v}. \tag{40}$$

Using equations (31, 37, 38, 39) and (40) , we get the following reduced form of equation (30) i.e.

$$mn \left[ (\frac{\partial \overrightarrow{u}}{\partial t}) + (\overrightarrow{u}.\overrightarrow{\nabla})\overrightarrow{u} \right] = \rho^{\star} \overrightarrow{\psi} + \overrightarrow{J} \times \overrightarrow{\psi} + \quad \triangle \overrightarrow{P} + \quad \overrightarrow{\nabla}.I, \tag{41}$$

which is the Boltzmann equation the case of dyonic plasma. In equation (41) $\rho^{\star}$ is the complex conjugate of the dyonic charge source density, $\vec{\psi}$ is complex vector field of dyon, $\overrightarrow{J}$ is current source density of dyon, $n_e$ is the number density of electric charge, $n_m$ is the number density of magnetic charge. Physical interpretation of equation (41) has thus been described due to the scattering between either purely electric or magnetic charge carriers. The equation (41) is thus described as the fluid equation of motion for generalized field of dyons (i.e. dyonic fluid) which is the modified form of second equation for magnetohydrodynamics (MHD) in case of dyonic plasma.

Third equation of magnetohydrodynamics (MHD) given by equation (24) is expressed as the generalized Ohm's law. We may write magnetohydrodynamics (MHD) equation (24) in presence of electric charge as

$$mn_e \left[ \frac{\partial u_e}{\partial t} + u_e \nabla u_e \right] \quad = \quad en_e[\vec{E} + \vec{v} \times \vec{H}] + \triangle \overrightarrow{P}_e. \tag{42}$$

Accordingly, the magnetohydrodynamics (MHD) equation (24) in presence of pure magnetic charge may be expressed as

$$mn_g \left[ \frac{\partial u_g}{\partial t} + u_g \nabla u_g \right] \quad = \quad gn_g[n_m \vec{H} - \vec{v} \times \vec{E}] + \triangle \overrightarrow{P}_g. \tag{43}$$

Multiplying equation (42) by $\frac{e}{m}$ and equation (43) by $\frac{ig}{m}$ and then subtracting , we get

$$en_e \frac{\partial u_e}{\partial t} - ign_g \frac{\partial u_g}{\partial t} = \frac{e^2 n_e}{m}[\vec{E} + \vec{v} \times \vec{H}] + \frac{e}{m} \triangle \overrightarrow{P}_e - i \frac{g^2 n_g}{m}[\vec{H} - \vec{v} \times \vec{E}] - i \frac{g}{m} \triangle \overrightarrow{P}_g, \tag{44}$$

where

$$
\begin{aligned}
j_e &= e n_e u_e; \\
j_m &= g n_g u_g.
\end{aligned} \tag{45}
$$

Using equations (44 - 45), we get

$$
\frac{\partial J}{\partial t} = \frac{1}{m}[e^2 n_e F_e - i g^2 n_g F_m] + \frac{1}{m}(e\triangle\overrightarrow{P_e} - ig\triangle\overrightarrow{P_m}) \tag{46}
$$

where $J = j_e - i j_g$ is used for the generalized current density for dyons, $F_e = e(\overrightarrow{E} + \overrightarrow{u} \times \overrightarrow{H})$ is the force acting on the electric charged particle and $F_m = g(\overrightarrow{H} - \overrightarrow{u} \times \overrightarrow{E})$ describes the force acting on the magnetic charged particle. As such, the equation (46) is the modified form of the third equation of magnetohydrodynamics (MHD) for the case of generalized fields of dyons (i.e. dyonic plasma). In the absence of the $\overrightarrow{H} = 0$ ($\overrightarrow{E} = 0$) all the three modified equations (28, 41 & 46) of magnetohydrodynamics (MHD) associated with dyonic plasma reduces to the usual differential equation of magnetohydrodynamics (MHD) in presence of electric ( magnetic) charge only. The equation (46) provides the combination of two Ohm's law. It is due to the mixed plasma of dyons as the consequence of presence of electric and magnetic charges on dyons. In this case we have considered low wavelength approximation. Consequently this case the electron - ion and magnetic monopole - magneto ions recombine due to short distance effects. If the plasma dynamics becomes too fast, resonances occur with the motions of individual particles which invalidate the MHD equations. Furthermore, effects, such as particle inertia and the Hall effect, which are not taken into account in the MHD equations, become important. Since MHD is a single fluid plasma theory, a single fluid approach is justified because the perpendicular motion is dominated by $\overrightarrow{E} \times \overrightarrow{H}$ drifts. For the case of slow plasma dynamics, the motions of the dyon and ion fluids become sufficiently different as single fluid approach is no longer tenable. This also occurs whenever the diamagnetic velocities, which are quite different for different plasma species, become comparable to the $\overrightarrow{E} \times \overrightarrow{H}$ velocity . Furthermore, effects such as plasma resistivity, viscosity, and thermal conductivity, which are not taken into account in the MHD equations, become important in this case.

# 4 Frequency of Dyonic Plasma

The case of dyonic plasma is not the case of single particle motions but rather collective motion of the various charge species of dyonic plasma (i.e. electric plasma and monopole plasma). So, the first, and most important is to discuss the electrostatic plasma oscillation responsible for plasma frequency. These oscillations occur when one of the species is displaced from the other.

For this we start with the equation of motion for a particle which has electric charge $e$ and mass $m$ in an electric field as

$$m\overrightarrow{\ddot{r}} = e\overrightarrow{E}. \tag{47}$$

where $\overrightarrow{\ddot{r}}$ denotes the acceleration. If $\overrightarrow{E} = \overrightarrow{E}\, e^{i(\overrightarrow{k}.\overrightarrow{r}-\omega t)}$ is expressed as an incident plane wave, then equation (47) reduces to

$$m\overrightarrow{\dot{r}} = -\frac{e\overrightarrow{E}}{i\omega}. \tag{48}$$

So the electric current source density is expressed as

$$\overrightarrow{j_e} = n_e e\overrightarrow{\dot{r}} = -\frac{n_e e^2\overrightarrow{E}}{im\omega} = \delta_e\overrightarrow{E} \tag{49}$$

where

$$\delta_e = -\frac{n_e e^2}{im\omega}. \tag{50}$$

Similarly for a particle with monopole of charge $g$ moving in a magnetic field, we have

$$\overrightarrow{j_m} = \delta_m\overrightarrow{H} \tag{51}$$

where

$$\delta_m = -\frac{n_g g^2}{im\omega}. \tag{52}$$

With the help of Maxwell's Dirac equation (1) and using the relations (45), (49- 52), we get

$$k^2 = \omega^2 - \frac{n_e}{m}qq* + \left(\frac{n_e eg}{m\omega}\right)^2 = \omega^2 - \omega_p^2 \tag{53}$$

where

$$\omega_p^2 = \left(\frac{n_e}{m}qq* - \left(\frac{n_e eg}{m\omega}\right)^2\right) \tag{54}$$

where $q = e - ig$ is the generalized charge of dyon. In equation (54) $\omega_p$ is the plasma frequency of the dyon, $q^\star$ is the complex conjugate of generalized charge $q$ of dyon and $k$ is known as the usual wave number. There is a different plasma frequency for each species. For $k$ to be real, only those generalized electromagnetic waves are allowed to pass, for which

$\omega > \omega_p$. At very high frequencies, $\omega = ck$, dyon can not respond fast enough, and plasma effects are negligible. Thus Plasma frequency sets the lower cuts for the frequencies of electromagnetic radiation that can pass through a plasma. The metals shine by reflecting most of light in visible range. The visible light can not pass through the metal because the plasma frequency of electrons in metal falls in ultraviolet region. For frequencies in ultra voilet region ($UV$), metals are transparent. The earth's ionosphere reflects radio waves in the same reason. The electron (monopole) densities at various heights in the ionosphere can be inered by studying the reflection of pulses of radiation transmitted vertically upwards. Also, the broadcast of various radio signals in communication on earth is possible only because of reflection from the ionosphere.

## 5 Magnetohydrodynamic (MHD) waves for dyons

Plasma is a complex fluid that support many plasma wave modes. Restoring forces include kinetic pressure and electromagnetic forces [12]. Let us investigate the small amplitude waves propagating through a spatially uniform MHD plasma. For this, we take the two cases. Case I- we define .

$$\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{H} = 0 \tag{55}$$

which describes the Ohm's law for the dynamics of electric charge. Similarly for case -II we have

$$\overrightarrow{H} - \overrightarrow{v} \times \overrightarrow{E} = 0 \tag{56}$$

which may be identified as the Ohm's law for free magnetic monopole. So, the force equation is expressed as

$$\rho \frac{\partial \overrightarrow{v}}{\partial t} + \rho(\overrightarrow{v}.\overrightarrow{\nabla})\overrightarrow{v} = (\overrightarrow{\nabla} \times \overrightarrow{H}) \times \overrightarrow{H} + (\overrightarrow{\nabla} \times \overrightarrow{E}) \times \overrightarrow{E} + \overrightarrow{\nabla} p \tag{57}$$

while the equation of state is given as

$$\overrightarrow{\nabla} p = -V_s^2 \overrightarrow{\nabla} \rho \tag{58}$$

where $V_s$ is the speed of the dyon. Substituting equation (58) into equation (57) we may write the force equation as

$$\rho \frac{\partial \overrightarrow{v}}{\partial t} + \rho(\overrightarrow{v}.\overrightarrow{\nabla})\overrightarrow{v} = (\overrightarrow{\nabla} \times \overrightarrow{H}) \times \overrightarrow{H} + (\overrightarrow{\nabla} \times \overrightarrow{E}) \times \overrightarrow{E} - V_s^2 \overrightarrow{\nabla} \rho. \tag{59}$$

Post multiplying vectorially the fourth Maxwell's Dirac equation (1) by $\overrightarrow{H}$ and rearranging the terms, we get,

$$\overrightarrow{j_e} \times \overrightarrow{H} = (\overrightarrow{\nabla} \times \overrightarrow{H}) \times \overrightarrow{H} - \frac{\partial \overrightarrow{E}}{\partial t} \times \overrightarrow{H}. \tag{60}$$

Similarly post multiplying vectorially the third Maxwell's Dirac equation (1) by $\overrightarrow{E}$, we get

$$\overrightarrow{j_m} \times \overrightarrow{E} = - (\overrightarrow{\nabla} \times \overrightarrow{E}) \times \overrightarrow{E} - \frac{\partial \overrightarrow{H}}{\partial t} \times \overrightarrow{E}. \tag{61}$$

Hence the equation (41) reduces to

$$\begin{aligned} mn\left[(\frac{\partial \overrightarrow{v}}{\partial t}) + (\overrightarrow{v}.\overrightarrow{\nabla})\overrightarrow{v}\right] =& en_e\overrightarrow{E} + gn_g\overrightarrow{H} \\ &+ (\overrightarrow{\nabla} \times \overrightarrow{H}) \times \overrightarrow{H} + (\overrightarrow{\nabla} \times \overrightarrow{E}) \times \overrightarrow{E} + \triangle P + \frac{\partial}{\partial t}(\overrightarrow{H} \times \overrightarrow{E}) + \overrightarrow{\nabla}.\overrightarrow{I}. \end{aligned} \tag{62}$$

With the help of equations (55), (56) and equation (1), we get

$$\frac{\partial \overrightarrow{H}}{\partial t} = \overrightarrow{\nabla} \times (\overrightarrow{v} \times \overrightarrow{H}) - \overrightarrow{j_m}; \tag{63}$$

$$\frac{\partial \overrightarrow{E}}{\partial t} = \overrightarrow{\nabla} \times (\overrightarrow{v} \times \overrightarrow{E}) - \overrightarrow{j_e}. \tag{64}$$

Applying the perturbation transformations about the equilibrium values i.e.

$$\begin{aligned} \rho &\longmapsto \rho_0 + \rho_1; \\ H &\longmapsto H_0 + H_1; \\ E &\longmapsto E_0 + E_1; \\ v &\longmapsto v_1; \end{aligned} \tag{65}$$

where $_0$ is the background density of the unperturbed fluid and $v_0 = 0$ (i.e. the fluid is at rest). As such, with the help of equation (65), we get the following reduced expressions for equations (28) and (62)

$$
\begin{aligned}
&\frac{\partial \rho_1}{\partial t} + \rho_0 \overrightarrow{\nabla}.\overrightarrow{v_1} = 0; \\
&\frac{\partial^2 \overrightarrow{v_1}}{\partial t^2} + V_s^2 \overrightarrow{\nabla}\left[-\overrightarrow{\nabla}.\overrightarrow{v_1}\right] + \frac{H_0}{\rho_0}\left[\overrightarrow{\nabla} \times \left\{\overrightarrow{\nabla} \times \left(\overrightarrow{v_1} \times \overrightarrow{H_0}\right) - \overrightarrow{j_m}\right\}\right] \\
&\qquad + \frac{E_0}{\rho_0}\left[\overrightarrow{\nabla} \times \left\{\overrightarrow{\nabla} \times \left(\overrightarrow{v_1} \times \overrightarrow{E_0}\right) - \overrightarrow{j_e}\right\}\right] = 0.
\end{aligned} \tag{66}
$$

Here we have used

$$
V_A = \frac{H_0}{(\rho_0)^{\frac{1}{2}}}; \quad V_B = \frac{E_0}{(\rho_0)^{\frac{1}{2}}}; \tag{67}
$$

$$
v_1(\overrightarrow{r}, t) = v_0 \exp i(\overrightarrow{k}.\overrightarrow{r} - \omega t); \tag{68}
$$

$$
\overrightarrow{\nabla} \longrightarrow i\overrightarrow{k} \quad and \quad \frac{\partial}{\partial t} \longrightarrow -i\omega. \tag{69}
$$

Hence the dispersion relation (66) takes the following form

$$
\begin{aligned}
&-\omega^2 v_1 + V_s^2(\overrightarrow{k}.\overrightarrow{v_1})k - \overrightarrow{V_A} \times \left[\overrightarrow{k} \times [\overrightarrow{k} \times (\overrightarrow{v_1} \times \overrightarrow{V_A}) + \frac{\overrightarrow{j_m}}{(\rho_0)^{\frac{1}{2}}}\right] \\
&\qquad - \overrightarrow{V_B} \times \left[\overrightarrow{k} \times [\overrightarrow{k} \times (\overrightarrow{v_1} \times \overrightarrow{V_B}) + \frac{\overrightarrow{j_e}}{(\rho_0)^{\frac{1}{2}}}\right] = 0.
\end{aligned} \tag{70}
$$

Expanding the equation (70) and using vector triple product, we get

$$
\begin{aligned}
-\omega^2 v_1 + &[V_s^2 + V_A^2 + V_B^2](\overrightarrow{k}.\overrightarrow{v_1})k \\
&+(\overrightarrow{k}.\overrightarrow{V_A})[(\overrightarrow{k}.\overrightarrow{V_A})\overrightarrow{v_1} - (\overrightarrow{V_A}.\overrightarrow{v_1})\overrightarrow{k} - (\overrightarrow{k}.\overrightarrow{v_1})\overrightarrow{V_A}] - \frac{\overrightarrow{V_A} \times (\overrightarrow{k} \times \overrightarrow{j_m})}{(\rho_0)^{\frac{1}{2}}} \\
&+(\overrightarrow{k}.\overrightarrow{V_B})[(\overrightarrow{k}.\overrightarrow{V_B})\overrightarrow{v_1} - (\overrightarrow{V_B}.\overrightarrow{v_1})\overrightarrow{k} - (\overrightarrow{k}.\overrightarrow{v_1})\overrightarrow{V_B}] - \frac{\overrightarrow{V_B} \times (\overrightarrow{k} \times \overrightarrow{j_e})}{(\rho_0)^{\frac{1}{2}}} = 0.
\end{aligned} \tag{71}
$$

It describes the case of a kind of dyonoacoustic wave. Substituting $\overrightarrow{k} \perp \overrightarrow{H_0}$ , $\overrightarrow{k} \perp \overrightarrow{E_0}$ and $\overrightarrow{j_m} \parallel \overrightarrow{V_A}$, $\overrightarrow{j_e} \parallel \overrightarrow{V_B}$ , we get

$$
\overrightarrow{k}.\overrightarrow{V_A} = \overrightarrow{k}.\overrightarrow{V_B} = 0 \tag{72}
$$

and

$$\overrightarrow{V_A}\times\overrightarrow{j_m}=\overrightarrow{V_B}\times\overrightarrow{j_e} \quad = \quad 0. \tag{73}$$

As such, we get the following expression for the dispersion relation

$$-\omega^2 v_1+[V_s^2+V_A^2+V_B^2](\overrightarrow{k}.\overrightarrow{v_1})k \quad = \quad 0. \tag{74}$$

The vector nature of the equation (74) requires that the perturbed fluid velocity $\overrightarrow{v_1}$ must be parallel to the propagation direction $\overrightarrow{k}$ so that $\overrightarrow{k}.\overrightarrow{v_1}=kv_1$. Thus the wave is longitudinal in nature and its dispersion relation becomes

$$v_\phi \quad =\tfrac{\omega}{k} \quad =V_s^2+V_A^2+V_B^2. \tag{75}$$

Therefore the dyon acoustic waves propagating with velocity $v_\phi$ in this case. This is known as the dyonoacoustic, dyonosonic or simply compressional wave which involves compression and rarefaction for the electromagnetic lines of force along with plasma oscillations.

## 6 Energy of dyons

The energy of the dyonic plasma is related with the dispersive properties of the wave oscillations. Starting from first principle for the electromagnetic energy density and taking into account the specific features of dispersive relations of electromagnetic waves, we may obtain the expression for electromagnetic energy density (namely the Poynting Theorem) [12]. From the third and fourth Maxwell's Dirac equation (1), we obtain

$$\overrightarrow{E}.(\overrightarrow{\nabla}\times\overrightarrow{H})-\overrightarrow{H}.(\overrightarrow{\nabla}\times\overrightarrow{E}) \quad = \quad \overrightarrow{E}.\frac{\partial\overrightarrow{E}}{\partial t}+\overrightarrow{H}.\frac{\partial\overrightarrow{H}}{\partial t}+\overrightarrow{j_e}.\overrightarrow{E}+\overrightarrow{j_m}.\overrightarrow{H}. \tag{76}$$

This equation may then be written as the conservation law of energy as

$$\frac{\partial W}{\partial t}+\overrightarrow{\nabla}.\overrightarrow{P} \quad = \quad 0 \tag{77}$$

where

$$\overrightarrow{P} \quad = \quad \overrightarrow{E}\times\overrightarrow{H} \tag{78}$$

is called the Poynting vector. The rate of change of the energy density $\frac{\partial W}{\partial t}$ is then defined as

$$\frac{\partial W}{\partial t} = \overrightarrow{E}.\frac{\partial \overrightarrow{E}}{\partial t} + \overrightarrow{H}.\frac{\partial \overrightarrow{H}}{\partial t} + \overrightarrow{j_e}.\overrightarrow{E} + \overrightarrow{j_m}.\overrightarrow{H} \tag{79}$$

so that we may obtain the energy density by taking time integration as

$$\begin{aligned} W(t) =& W_0(t) + \int_{t_0}^{t} dt[\overrightarrow{E}.\frac{\partial \overrightarrow{E}}{\partial t} + \overrightarrow{H}.\frac{\partial \overrightarrow{H}}{\partial t} + \overrightarrow{j_e}.\overrightarrow{E} + \overrightarrow{j_m}.\overrightarrow{H}] \\ =& W_0(t) + \left[\frac{E_0^2 + H_0^2}{2}\right] + \int_{t_0}^{t} dt(\overrightarrow{j_m}.\overrightarrow{H} + \overrightarrow{j_e}.\overrightarrow{E}) \end{aligned} \tag{80}$$

where $W_0(t)$ is the energy density at reference point $t_0$. The quantity $(\overrightarrow{j_m}.\overrightarrow{H} + \overrightarrow{j_e}.\overrightarrow{E})$ is the rate of change of kinetic energy density of the dyon. This can be seen by taking the dot product of force equation which is taking this form with $\overrightarrow{v}$

$$m\overrightarrow{v}.\frac{d\overrightarrow{v}}{dt} = \overrightarrow{v}.(\overrightarrow{F_e} + \overrightarrow{F_m}) = \overrightarrow{v}.[e(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{H}) + g(\overrightarrow{H} - \overrightarrow{v} \times \overrightarrow{E})] \tag{81}$$

which may also be written as

$$\frac{d}{dt}(\frac{1}{2}mv^2) = e\overrightarrow{E}.\overrightarrow{v} + g\overrightarrow{H}.\overrightarrow{v}. \tag{82}$$

Since this is the rate of change of kinetic energy of a single dyon, the rate of change of the kinetic energy density $T$ for the entire system of dyons is found by summing over the energies of the individual dyons i.e.

$$\frac{d}{dt}(T) = \sum_i \int dv f_i(e_i\overrightarrow{E}.\overrightarrow{v} + g_i\overrightarrow{H}.\overrightarrow{v}) = \overrightarrow{E}.\overrightarrow{j_e} + \overrightarrow{H}.\overrightarrow{j_m} = Re(\overrightarrow{J}.\overrightarrow{\psi}^{\star}). \tag{83}$$

This shows that positive value of $(\overrightarrow{E}.\overrightarrow{j_e} + \overrightarrow{H}.\overrightarrow{j_m})$ corresponds to the increases the kinetic energy of dyons whereas the negative value of $(\overrightarrow{E}.\overrightarrow{j_e} + \overrightarrow{H}.\overrightarrow{j_m})$ corresponds to the decrease of kinetic energy of dyons. The latter situation is possible only if the dyon starts working with a finite initial kinetic energy density. Since $(\overrightarrow{E}.\overrightarrow{j_e} + \overrightarrow{H}.\overrightarrow{j_m})$ accounts for the changes in the dyon kinetic energy density, $W$ must be the sum of the generalized electromagnetic field density and the particle energy density. In our case, we have considered a fluid element of dyonic plasma for which the overall charge is taken to be neutral. So, an external electromagnetic field cannot cause motion of a fluid element as a whole, but will sets up currents due to the motion of opposite charges in opposite directions. Due to these currents, an external electromagnetic field exerts a force on the fluid element and changes its direction of motion. Here we have described the motion of plasma oscillations for two different fluids associated with the electric and magnetic charges or a composite system of dyons.

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